A COMPLETE DISQUISITION OF VARIOUS PARAMETERS ON THE BENDING ANALYSIS OF FUNCTIONALLY GRADED NANOBEAMS

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ABSTRACT*: In this research, a complete investigation on the bending analysis of nonlocal functionally graded nanobeams is performed. The material properties of functionally graded nanobeam are distributed continuously through thickness direction according to power law function. The size dependency of nanobeam is described by the differential constitutive equation of Eringen. The equilibrium equations have been derived using the principle of the minimum total potential energy, under the assumptions of the classical beam theories such as Euler-Bernoulli and Timoshenko. The finite element method is employed to discretize the model and obtain a numerical approximation of equilibrium equations. The model has been verified with peer published works and found an excellent agreement with them. Finite element numerical results are presented in both tabular and graphical forms to figure out the effects of different material distribution profile, nonlocal effect, transverse shear effect, slenderness ratios and boundary conditions on the bending characteristics of nanobeams*

KEYWORDS*: Functionally graded material, power-law function, finite element method, Euler-Bernoulli beam theory, Timoshenko beam theory*

1 INTRODUCTION

Nanotechnology is essentially concerned with the fabrication of FGM and designing structures at a nanoscale, which provides a new class of materials with radical properties and improved functionality. nanobeams are one of the most important nanostructures used in systems and nanodevices, thence, the understanding of its mechanical behavior is inescapable for developing of such structures. A brief review of papers devoted to the current study is presented as follows: Hamed et al. [1] derived equations for local and nonlocal beam to investigate the mechanical responses of perforated nanobeams on the static bending and buckling by considering all the boundary conditions. Belarbi et al. [2] proposed a nonlocal finite element formulation for flexural behavior of nanocomposite nanobeams with a new shear deformation theory. A new polynomial higher-order shear deformation theory was introduced and developed by Ziou et al. [3] for static analysis of functionally graded material (FGM) beams. The developed theory does not require shear correction factor and satisfies the stress-free boundary conditions, such that the transverse shear stress varies parabolically through the beam thickness. Sayyad and Ghugal [4] presented a unified formulation of twenty-one nonlocal beam theories to study the bending, buckling and free vibration behavior of FG nanobeams by using the nonlocal differential constitutive relations of Eringen. Merzouki et al. [5] developed two-variable-based trigonometric SDT for bending, buckling, and free vibration analysis of nanobeams. A complete investigation on the significance of the transverse shear for the buckling analysis of FGM beam was performed by Ziou et al. [6]. Two separate finite element formulations were developed; one based on Euler-Bernoulli theory and the other one on Timoshenko beam theory. The results showed that the transverse shear should be considered to better predict the critical loads in FGM beam type structures. Apuzzo et al. [7] investigated the size-dependent bending behavior of nanobeams by using the modified nonlocal strain gradient elasticity theory by considering all the boundary conditions. Hashemian et al. [8] proposed a size-dependent beam models using different beam theories and nonlocal strain gradient theory incorporating surface effects in order to study bending and buckling behaviors of nanobeams. Nikam and Sayyad [9] presented a nonlocal beam theories using unified formulation to examine the bending, buckling, and vibration responses of simply supported nanobeams. Aria and Friswell [10] exploited a nonlocal strain-driven elasticity to probe natural frequency and buckling of FGM nanobeams in which the material properties were assumed to obey a power-law function. Eltaher et al. [11] presented coupled influences of nonlocal size scale and surface energy properties on bending and vibrational behaviors of piezoelectric thin nanobeams. Barretta et al. [12] employed the stress-driven nonlocal elasticity approach with the Euler-Bernoulli FGM nanobeam model and proved the inapplicability of the strain-driven integral model on the solution. Exact solution for bending analysis of FGM nanobeams was presented. Eltaher et al. [13] investigated the static-buckling behavior of functionally graded (FG) nanobeams numerically; they used the nonlocal elasticity theory introduced by Eringen. Simsek and Yurtcu [14] examined analytically bending and buckling of FG nanobeam based on the nonlocal Timoshenko and Euler–Bernoulli beam theories.

In the present work, a nonlocal finite element model is developed to study the static bending of FGM nanobeams. Two separate engineering beam theories are presented with details, one missing transverse shear (NEBT), and the other take it into account (NTBT). The nonlocal elastic behavior is described by the differential constitutive model of Eringen. The material properties of FG nanobeams are assumed to vary through the thickness according to the power law. The effects of nonlocal parameter, transverse shear effect, slenderness ratio, various material compositions and boundary conditions on the static behavior of the FGM nanobeams are discussed.

To our best knowledge, there is no reported and detailed work on the analysis of nonlocal functionally graded nanobeams including the effect of power law index, nonlocal effect, transverse shear effect, slenderness ratios and boundary conditions in the literature. It is believed that the presented results will provide a reference with which other researchers can compare their results.

2 MATHEMATICAL FORMULATIONS

2.1 Material properties

An FGM is defined to be a material which has a non-uniform gradation in the thickness direction. Topmost surface consists of only metal and bottom surface is only ceramic, In between a mixture of the two materials. The material variation is dictated by a parameter, “p”. At p = 0 the beam is a fully metal beam while at p = ∞ the beam is fully ceramic. The typical material property P is varied through the nanobeam thickness according to power law distribution of the volume fraction of the constituents.

$$P(z)=\left(P_m-P_c\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p+P_c \qquad (1)$$

$$V_f(z)=\left(\frac{z}{h}+\frac{1}{2}\right)^p \qquad (2)$$

Where *P* denotes the material properties of FGM beam (ie., modulus of elasticity E and shear modulus G). The subscripts *c* and *m* refer to the ceramics and metal respectively; the parameter p is known as the power law index of FGM. Also, h is the thickness of beam and z is the coordinate in the thickness direction. The variation of the volume fraction of the ceramic constituent along the thickness of the FGM beam is plotted in Fig. 1 with respect to various values of p. The effect of Poisson’s ratio on the deformation is much less than Young’s modulus and that has been confirmed by Ziou et al. [15], Delale and Erdogan [16] with an energetic method. Therefore, the same Poisson’s ratio is adopted for both materials in the present analysis.

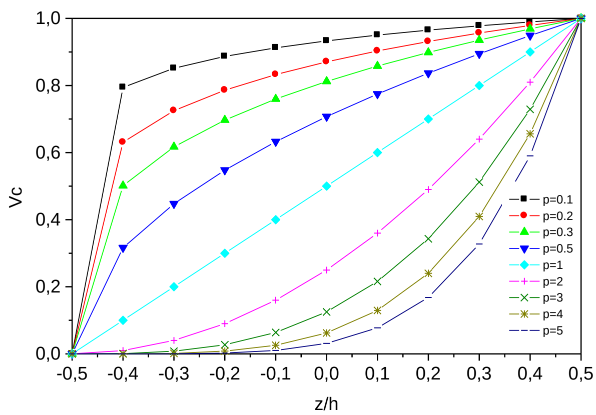

Fig. 1 The variation of the volume fraction of the ceramic constituent along the thickness

2.2 Nonlocal Elasticity Theory

In the classical (local) elasticity theory, the stress at a point depends only on the strain at the same point, whereas in the nonlocal elasticity theory the stress at a point is a function of strains at all points in the continuum (Eringen, [17]). Therefore, the nonlocal stress tensor at point x is expressed by Reddy et al., [18] as:

$$\sigma=\int_V K\left(\left|x'-x\right|,\tau\right)T\left(x'\right)dx',\ T(x)=C(x):\varepsilon(x) \qquad (3)$$

The above original designs are transformed individually into their corresponding generalized chains (kinematic chains). The generalized chain will be involved in various types of members (edges) and joints (vertices, or said kinematic pairs) for all possible assembly in the following steps.

Where $T(x')$ is the classic macroscopic stress tensor at point x, $\varepsilon(x)$ is the strain tensor, $C(x)$ is the fourth-order elasticity tensor, $K(|x'-x|,\tau)$ is the nonlocal modulus, $|x'-x|$ is the Euclidean distance, and $\tau = e_0 a / l$ is defined as small scale factor. e_0 is a constant to adjust the model to match the reliable results by experiments or other models, a is internal characteristic length (e.g. lattice parameter, C–C bond length, granular distance, crack length, wavelength), and l is the external length.

A simplified form of the constitutive relation may be represented as:

$$\left(1-\tau^2 l^2 \nabla^2\right)\sigma = t \tag{4}$$

Where ∇^2 is the Laplacian operator. The nonlocal behavior for a beam structure can be neglected in the thickness direction. Therefore, Eq. (4) takes the following form:

$$\sigma_{xx} - \mu \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E(z)\varepsilon_{xx} \tag{5}$$

$$\sigma_{xz} - \mu \frac{\partial^2 \sigma_{xz}}{\partial x^2} = G(z)\gamma_{xz} \tag{6}$$

The classical local elasticity theory is recovered from Eqs. (5 and 6) when $\mu = a^2 e_0^2 = 0$

2.3 Nonlocal Beam Theories

2.3.1 Nonlocal Timoshenko Beam Theory (NTBT)

Based on Timoshenko beam theory, the kinematic relations are given by.

$$u(x,z) = u_0(x) - z\varphi(x) \tag{7-a}$$

$$w(x,z) = w_0(x) \tag{7-b}$$

Where $(-)_0$ denotes the displacements of the beams axis. φ is the total bending rotation of the cross-section

The axial and transverse strains are deduced from Eqs. (7) as:

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x} - z\frac{\partial \varphi}{\partial x} = \varepsilon_{xx}^0 - z\kappa_{xx}^0 \tag{8}$$

$$\gamma_{xz} = \frac{\partial w_0}{\partial x} - \varphi \tag{9}$$

ε_{xx}^0 is the extensional strain and κ_{xx}^0 is the bending strain.

An elastic body is in equilibrium, if the total virtual work done by actual external and internal forces is zero

$$\delta\pi = \delta\left(W_{\text{int}} - W_{ext}\right) = 0 \tag{10}$$

The virtual strain energy W_{int} and the virtual work done by external loading W_{ext} are given respectively by:

$$\delta W_{\text{int}} = \int_V \left(\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{xx}\delta\gamma_{xz}\right) dV = \int_0^L \left(N\delta\varepsilon_{xx}^0 + M\delta\kappa_{xx}^0 + Q\delta\gamma_{xz}\right) dx \tag{11}$$

$$\delta W_{ext} = \int_0^L \bar{P}\left(\frac{\partial \delta u_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial \delta w_0}{\partial x}\right) dx + \left[\bar{V}\delta w_0 - \bar{M}\delta\varphi\right]_0^L + \int_0^L \left(q\delta w_0 + f\delta u_0\right) dx \tag{12}$$

$f(x)$ and $q(x)$ are the axial and transverse distributed loads, $\bar{P}$ is the applied axial compressive force, $\bar{V}$ is the external shear force, $\bar{M}$ is the external bending moment.

Substituting Eqs. (11) and (12) into Eq. (10), integrating by parts and setting the coefficient of the admissible displacement and rotation to zero yields:

$$\frac{\partial N}{\partial x} + f = 0 \tag{13-a}$$

$$\frac{\partial^2 M}{\partial x^2} - Q = 0 \tag{13-b}$$

$$\frac{\partial Q}{\partial x} + q - \bar{P}\frac{\partial^2 w}{\partial x^2} = 0 \tag{13-c}$$

By using Eqs. (3), (4), (8), (9) and (13), forces and moment resultant of the nonlocal FGM Timoshenko beam in terms of displacement can be obtained as follows:

$$N - \mu\frac{\partial^2 N}{\partial x^2} = \hat{D}_a\frac{\partial u_0}{\partial x} - \hat{D}_{ab}\frac{\partial \varphi}{\partial x} \tag{14}$$

$$M - \mu\frac{\partial^2 M}{\partial x^2} = \hat{D}_{ab}\frac{\partial u_0}{\partial x} - \hat{D}_b\frac{\partial \varphi}{\partial x} \tag{15}$$

$$Q - \mu\frac{\partial^2 Q}{\partial x^2} = \kappa_s\hat{D}_s\left(\frac{\partial w_0}{\partial x} - \varphi\right) \tag{16}$$

$\hat{D}_a$ Is the axial stiffness, $\hat{D}_b$ is the bending stiffness, $\hat{D}_{ab}$ is the coupling axial-bending stiffness and $\hat{D}_s$ is the shear stiffness. These stiffness coefficients can be calculated by:

$$\left(\hat{D}_a, \hat{D}_{ab}, \hat{D}_b\right) = b\int_{-h/2}^{+h/2} E(z)\left(1, z, z^2\right) dz \tag{17}$$

$$\hat{D}_s = b\int_{-h/2}^{+h/2} G(z) dz \tag{18}$$

The nonlocal normal force can be obtained by differentiating Eq. (13-a) and substituting the result into Eq. (14) as following:

$$N = A_{xx}\frac{\partial u_0}{\partial x} - B_{xx}\frac{\partial \varphi}{\partial x} - \mu\frac{\partial f}{\partial x} \qquad (19)$$

The nonlocal bending moment expression is given by substituting the second derivative of M by eliminating the shear force Q between Eqs. (13-c) and (13-b)

$$M = B_{xx}\frac{\partial u_0}{\partial x} - D_{xx}\frac{\partial \varphi}{\partial x} + \mu\left(\bar{P}\frac{\partial^2 w_0}{\partial x^2} - q\right) \qquad (20)$$

The nonlocal shear force need to be determined by substituting the second derivative of Q from Eq. (13-c) into Eq. (16) as follows:

$$Q = \kappa_s A_{xz}\left(\frac{\partial w_0}{\partial x} - \varphi\right) + \mu\left(\bar{P}\frac{\partial^3 w_0}{\partial x^3} - \frac{\partial q}{\partial x}\right) \qquad (21)$$

2.3.2 Nonlocal Timoshenko Beam Theory (NTBT)

Based on Euler-Bernoulli beam theory, the kinematic relations are listed as follows:

$$u(x,z) = u_0(x) - z\frac{\partial w_0(x)}{\partial x} \qquad (22\text{-a})$$

$$w(x,z) = w_0(x) \qquad (22\text{-b})$$

The nonzero strain according to Euler-Bernoulli beam theory can be expressed by:

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2} = \varepsilon_{xx}^0 - z\kappa_{xx}^0 \qquad (23)$$

In accordance with the same process that was applied for Timoshenko beam theory, the nonlocal axial normal force and bending moment can be found according the following equations:

$$N = A_{xx}\frac{\partial u_0}{\partial x} - B_{xx}\frac{\partial^2 w_0}{\partial x^2} - \mu\frac{\partial f}{\partial x} \qquad (24)$$

$$M = B_{xx}\frac{\partial u_0}{\partial x} - D_{xx}\frac{\partial^2 w_0}{\partial x^2} + \mu\left(\bar{P}\frac{\partial^2 w_0}{\partial x^2} - q\right) \qquad (25)$$

3 NUMERICAL RESULTS AND DISCUSSION

Model validation was firstly presented to verify the efficiency of the present formulations with peer published works. For this purpose of verification, the non-dimensional maximum deflection for completely homogenous ceramic nanobeam obtained from the present model is compared with those obtained from Simsek and Yurtcu [14] and tabulated in Table 1. The geometrical and material properties of a nonlocal FGM beam used in this subsection according to Simsek and Yurtcu [14]. The non-dimensional maximum deflection is defined as following:

$$w_{max} = 100 \times \delta_{max} \times \frac{EI}{q_0 L^4} \qquad (26)$$

As can be observed from the table 1, the predicted results given by the present formulations are in good agreement with those reported by Simsek and Yurtcu [14] for all nonlocal parameter μ. It is interesting to note that the results of Simsek and Yurtcu [14] are evaluated based on analythical solutions under the assumption of Fourier series.

3.1 Slenderness Ratio Effect

The effects of geometrical and nonlocal parameters on the non-dimensional maximum deflection of simply supported nanobeam (S-S), using NCBT and NTBT are illustrated in Table 2 and Fig. 2.

It is worth noting that all the values of the deflection increase with an increase of the nonlocal parameter. Furthermore, the difference between the values predicted by NEBT and NTBT theories get more pronounced at small values of length to thickness ratio (short nanobeams), which confirms the importance of using the NTBT theory for non-slender beam, as the slenderness ratio has no significant effect on non-dimensional maximum deflection for NEBT theory.

Similar observations have been shown using plots in Fig. 2, the difference between the values predicted by NEBT and NTBT theories decreases with an increase of the nonlocal parameter. The maximum difference is observed for μ=0.

3.2 Transverse Shear Effect

Table 2 and Fig. 3 illustrate the effects of slenderness ratio and nonlocal parameter on the deflection for simply supported nanobeam using two formulations.

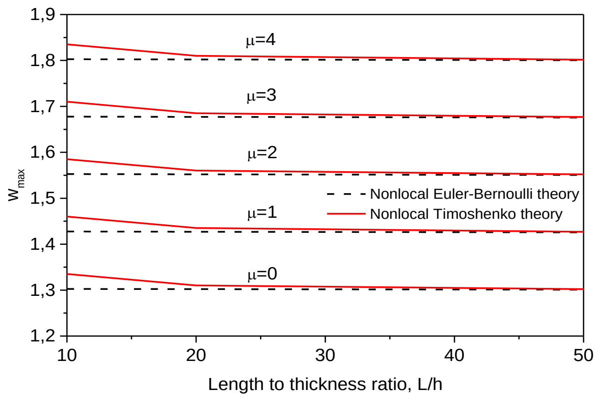

Fig. 3 The variation of non-dimensional deflection with respect to μ and L/h (S-S)

For a constant nonlocal parameter, the non-dimensional deflection does not match for lower slenderness ratio, which emphasizes the effect of transverse shear that increases the flexibility of thick beams and therefore reduces the deflection. By comparing the two formulations, the deflection of the Timoshenko nanobeam is higher than the Euler-Bernoulli nanobeam, due to missing the shear effect in the later. The values obtained with nonlocal Timoshenko beam theory converged to those obtained by nonlocal Euler-Bernoulli theory after L/h=20 and the two curves coincide after L/h=50.

3.3 Boundary Condition Effect

In this section, finite element solutions for bending analysis of nanobeams subjected to uniform load are carried out. Material properties of FGM constituents used here according to Alshorbagy et al. [19]. Different boundary conditions are considered. The obtained results are evaluated with the existing solutions and an excellent agreement is observed for all nonlocal parameter values and all boundary conditions. It is important to note that the NEBT model is used only in this section,

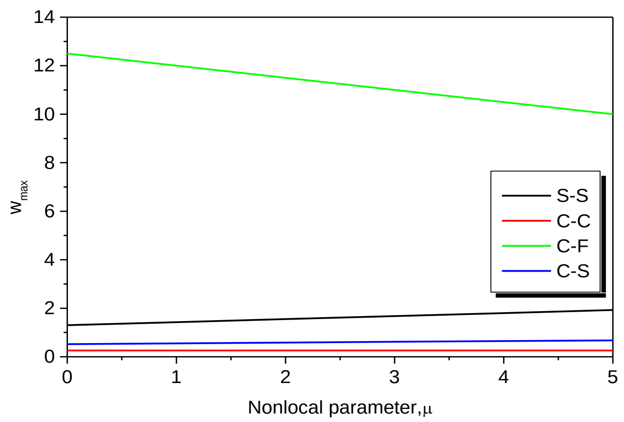

Fig. 4 The effect of nonlocal parameter on non-dimensional maximum deflection for different boundary conditions

From table 3 and figure 4, it can be observed that with raising the nonlocal parameter, the non-dimensional maximum deflection increases for S-S and C-S end conditions, and decrease for C-F counterpart. Furthermore, the nonlocal parameter has no significant effect on non-dimensional maximum deflection for C-C end condition. Similar results have been shown using plots in Fig. 4, the nonlocal parameter is more prominent with C-F end conditions compared to S-S, C-C and C-S counterparts. By the way, the values of the non-dimensional maximum deflection with C-F nanobeam are higher than those for the other boundary conditions. Accordingly, C-F boundary conditions are more affected by the nonlocal parameter compared to the other boundary conditions. In other words, the influence of nonlocal parameter decreases as the rigidity of the nanobeam increases.

3.4 Power law index effect

After validation of the homogenous nanobeam, a systematic analysis of nonlocal FGM nanobeam should be conducted to establish the trend of nonlocality, slenderness ratio, material distribution profile on the bending behavior. An FGM nanobeam is considered for the study, the topmost surface of the nanobeam is pure metal, whereas the bottom surface is only ceramic. The material properties of steel and ceramic are given in table 4.

The influence of power-law index, nonlocality and length-to-thickness ratio on the non-dimensional deflection for S-S FGM nanobeam is exhibited in table 5 and figure 5.

As can be observed, the maximum deflection values are obtained for full metal nanobeams (p=0). As the power-law index increases, the deflection shows a downward trend for all nonlocal parameters and all length-to-thickness ratios. As mentioned earlier, the values of the deflection increase with an increase of the nonlocal parameter by using both beam formulations, slenderness ratio has no significant effect on non-dimensional maximum deflection for NEBT theory.

Table 1 Non-dimensional maximum deflections of homogenous ceramic nanobeam for S-S beam subjected to uniform load

L/h	μ	NEBT		NTBT	
		[14]	Present	[14]	Present
10	0	1.3020	1.3026	1.3345	1.3351
	1	1.4270	1.4276	1.4621	1.4601
	2	1.5520	1.5526	1.5897	1.5851
	3	1.6770	1.6777	1.7173	1.7102
	4	1.8020	1.8027	1.8449	1.8352

Table 2 Effects of (L/h) and nonlocal parameter (μ) on non-dimensional maximum deflection for (S-S) nanobeam

L/h	Theories	μ=0	μ=1	μ=2	μ=3	μ=4
10	NCBT	1,3026	1,4276	1,5526	1,6777	1,8027
	NTBT	1,3351	1,4601	1,5851	1,7102	1,8352
	% diff.	2,4343	2,2259	2,0503	1,9004	1,7709
20	NCBT	1,3021	1,4271	1,5521	1,6771	1,8021
	NTBT	1,3102	1,4352	1,5602	1,6852	1,8102
	% diff.	0,6182	0,5644	0,5192	0,4807	0,4475
50	NCBT	1,3009	1,4258	1,5507	1,6757	1,8005
	NTBT	1,3022	1,4271	1,552	1,6768	1,8017
	% diff.	0,0998	0,0911	0,0838	0,0656	0,0666

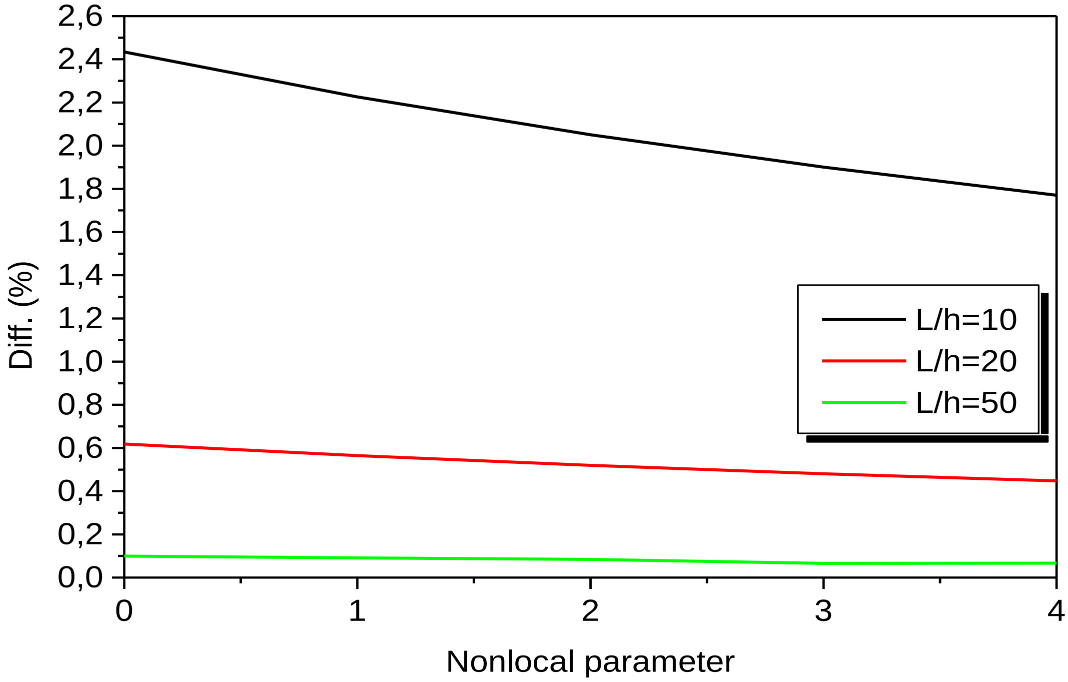

Fig. 2 The effect of nonlocal parameter on the per cent difference in non-dimensional deflection

Table 3 Non-dimensional maximum deflection validation with different boundary conditions

BCs	μ=0		μ=1		μ=2		μ=3		μ=4		μ=5	
	[19]	Present	[19]	Present	[19]	Present	[19]	Present	[19]	Present	[19]	Present
S-S	1.302	1.302	1.427	1.427	1.552	1.552	1.677	1.677	1.802	1.802	1.927	1.927
C-C	0.2604	0.2604	0.2604	0.2604	0.2604	0.2604	0.2604	0.2604	0.2604	0.2604	0.2604	0.2604
C-F	12.50	12.50	12.00	12.00	11.50	11.50	11.00	11.00	10.50	10.50	10.00	10.00
C-S	0.5416	0.5208	0.5769	0.5521	0.6122	0.5833	0.6480	0.6164	0.6840	0.6458	0. 72	0.6771

Table 4. Material properties of FGM constituents

Properties	Unit	Steel	Ceramic
E	TPa	0.25	1
v	/	0.3	0.3

Table 5. Non-dimensional maximum deflection of the nonlocal simply supported FGM nanobeam under uniform load

L/h	p	Nonlocal parameter μ									
		0		0.5		1		1.5		2	
		NEBT	NTBT	NEBT	NTBT	NEBT	NTBT	NEBT	NTBT	NEBT	NTBT
10	0	5,2083	5,3383	5,4583	5,5883	5,7083	5,8382	5,9584	6,0883	6,2083	6,3383
	1	2,3674	2,4194	2,4811	2,5331	2,5947	2,6467	2,7084	2,7604	2,8221	2,8741
	2	2,0827	2,4783	2,1407	2,1841	2,2388	2,2822	2,3369	2,3802	2,435	2,4783
	3	1,8853	1,9253	1,9759	2,0158	2,0663	2,1063	2,1568	2,1968	2,2473	2,2873
	4	1,7865	1,8248	1,8723	1,9105	1,9581	1,9963	2,0438	2,0821	2,1296	2,1678
30	0	5.2083	5.2306	5.4583	5.4813	5.7083	5.7317	5.9584	5.9820	6.2083	6.2324
	1	2.3674	2.3759	2.4811	2.4898	2.5947	2.6036	2.7084	2.7174	2.8221	2.8312
	2	2.0427	2.0499	2.1407	2.1481	2.2388	2.2463	2.3369	2.3445	2.4350	2.4427
	3	1.8853	1.8920	1.9759	1.9826	2.0663	2.0732	2.1568	2.1638	2.2473	2.2544
	4	1.7865	1.7929	1.8723	1.8788	1.9581	1.9646	2.0438	2.0505	2.1296	2.1364
100	0	5,2083	5,2096	5,4583	5,4596	5,7083	5,7096	5,9584	5,9596	6,2083	6,2097
	1	2,3674	2,3671	2,4811	2,4807	2,5947	2,5943	2,7084	2,708	2,8221	2,8216
	2	2,0427	2,0423	2,1407	2,0404	2,2388	2,2384	2,3369	2,3365	2,435	2,4345
	3	1,8853	1,885	1,9759	1,9755	2,0663	2,066	2,1568	2,1564	2,2473	2,2469
	4	1,7865	1,7863	1,8723	1,872	1,9581	1,9578	2,0438	2,0435	2,1296	2,1292

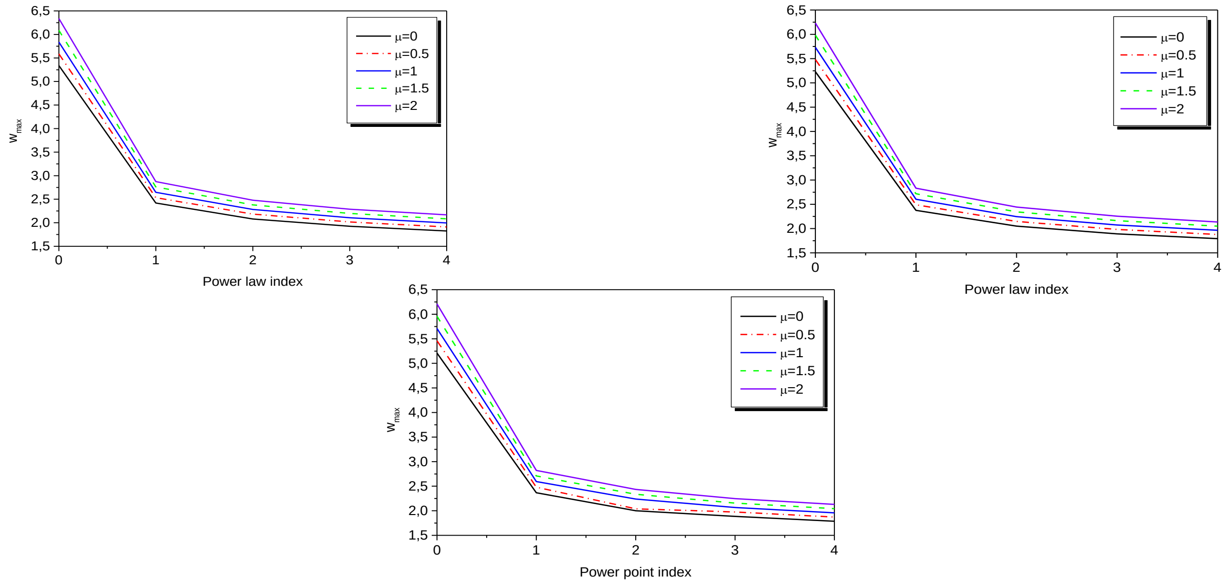

Fig. 5. The variation of non-dimensional deflections with respect to power-law index and nonlocal parameter for S-S FGM nanobeams (TBT): a) L/h=10, b) L/h=30, c) L/h=100

4 CONCLUSION

In this research, a total disquisition on the significance of slenderness ratio, nonlocality parameter, transverse shear, boundary conditions and power law index for the bending analysis of nanobeams made of FGM is carried out. To accomplish this, classical beam theories such as Euler-Bernoulli and Timoshenko are used. It is shown through numerical results that:

The values of deflection increase with an increase of the nonlocal parameter.

The difference between the values predicted by the two theories get more pronounced for short nanobeams, which confirms the importance of using the NTBT theory for non-slender beam

The slenderness ratio has no significant effect on non-dimensional maximum deflection for NEBT theory.

The difference between the values predicted by the used theories decreases with an increase of nonlocal parameter. The maximum difference is observed for μ=0.

For a constant nonlocal parameter, the non-dimensional deflection does not match for lower slenderness ratio, which emphasizes the effect of transverse shear that increases the flexibility of thick beams and therefore reduces deflection. By comparing the two formulations, the deflection of Timoshenko nanobeam is higher than Euler-Bernoulli nanobeam, due to missing a shear effect in the later.

With raising the nonlocal parameter, the non-dimensional maximum deflection increases for S-S and C-S end conditions, and decrease for C-F counterpart. Accordingly, the nonlocal parameter has no significant effect on non-dimensional maximum deflection for C-C end condition.

The nonlocal parameter is more prominent with C-F end conditions compared to S-S, C-C and C-S counterparts. By the way, the values of the non-dimensional maximum deflection with C-F nanobeam are higher than those for the other boundary conditions.

C-F boundary conditions are more affected by the nonlocal parameter compared to the other boundary conditions. In other words, the influence of nonlocal parameter decreases as the rigidity of the nanobeam increases. The results also show that the maximum deflection values are obtained for full metal nanobeams (p=0). As the power-law index increases, the content of ceramic (metal) in FGM increases.

5 REFERENCES

Hamed M., Mohamed, N. & Eltaher, M.A. (2022). Stability buckling and bending of nanobeams including cutouts. Engineering with Computers, 38: 209–230.

Belarbi M. O., Houari M., Daikh, A. A., Garg, A., Merzouki, T., Chalak, H.D., Hirane H. (2021). Nonlocal finite element model for the bending and buckling analysis of functionally graded nanobeams using a novel shear deformation theory. Composite Structures, 264: 113712.

Ziou, H., Guenfoud H., Guenfoud M. (2021). A Simple Higher-order Shear Deformation Theory for Static Bending Analysis of Functionally Graded Beams. Jordan Journal of Civil Engineering, 15 (2), 209-224.

Sayyad A. & Ghugal Y. (2020). Bending, Buckling and Free Vibration Analysis of Size-Dependent Nanoscale FG Beams Using Refined Models and Eringen’s Nonlocal Theory. International Journal of Applied Mechanics, 49: 251-267.

Merzouki T. & Houari M. & Haboussi M. & Bessaim A. & Ganapathi M. (2020). Nonlocal strain gradient finite element analysis of nanobeams using two-variable trigonometric shear deformation theory. Engineering with Computers.

Ziou, H., Guenfoud H., Guenfoud M. (2020). Buckling analysis behavior of functionally graded beams. Jordan Journal of Civil Engineering, 14 (3), 347-358.

Apuzzo A., Barretta R., Faghidian S.A., Luciano R., De Sciarra FM. (2019). Nonlocal strain gradient exact solutions for functionally graded inflected nano-beams. Composites Part B: Engineering, 164: 667–674.

Hashemian M. & Foroutan S. & Toghraie D. (2019). Comprehensive beam models for buckling and bending behavior of simple nanobeam based on nonlocal strain gradient theory and surface effects. Mechanics of Materials, 139: 103209.

Nikam R. & Sayyad A. (2019). A unified nonlocal formulation for bending, buckling and free vibration analysis of nanobeams. Mechanics of Advanced Materials and Structures, 27: 807-815.

Aria A.I., Friswell M.I. (2019). A Nonlocal Finite Element Model for Buckling and Vibration of Functionally Graded Nanobeams. Composites Part B Engineering. 166: 233-246.

Eltaher, MA., Omar, FA., Abdalla, WS., Gad, EH. (2019). Bending and Vibrational Behaviors of Piezoelectric Nonlocal Nanobeam Including Surface Elasticity. Waves Random Complex Media, 29(2), 264-280.

Barretta R. & Canadija M. & Feo L. & Luciano R. & Marotti de Sciarra F. & Penna R. (2018). Exact solutions of inflected functionally graded nano-beams in integral elasticity. Composites Part B Engineering, 142: 273-286.

Eltaher, M.A., Emam, S., Ibrahim, F. (2013). Static and stability analysis of nonlocal functionally graded nanobeams. Composite Structures, 96, 82–88.

Simsek, M., Yurtcu H.H. (2013). Analytical Solutions for Bending and Buckling of Functionally Graded Nanobeams Based on the Nonlocal Timoshenko Beam Theory. Composite Structures, 97, 378-386.

Ziou, H., Guenfoud H., Guenfoud M. (2016). Numerical Modelling of a Timoshenko FGM Beam Using the Finite Element Method. International Journal of Structural Engineering, 7(3), 239-261.

Delale, F., Erdogan, F. (1983). The Crack Problem for a Nonhomogeneous Plane, Journal of Applied Mechanics, 50(3), 609-614.

Eringen, A.C. (1983). On Differential Equations of Nonlocal Elasticity and Solutions of Screw Dislocation and Surface Waves. Journal Applied Physics, 54 (9), 4703- 4710.

Reddy, JN., Pang, SD. (2007). Nonlocal Theories for bending buckling and vibration of beams. International Journal of Engineering Science, 45, 288-307.

Alshorbagy E. A., Eltaher, M. A. and Mahmoud, F. F. (2013). Static Analysis of Nanobeams Using Nonlocal FEM. Journal of Mechanical Science and Technology, 27 (7), 2035-2041.

6 NOTATION

FGM - Functionally Graded Material
C-C - Clamped-Clamped
C-F - Clamped-Free
C-S - Clamped-Simply supported
NEBT - Nonlocal Euler-Bernoulli Theory
NTBT - Nonlocal Timoshenko Theory
S-S - Simply Supported